OPCIÓN A

1.A.- Sea **f(x)** una función derivable en **(0 , 1)** y continua en **[0 , 1]**, tal que **f(1) = 0** y $\int_0^1 2xf'(x)\,dx = 1$. Utilizar la fórmula de integración por partes para calcular $\int_0^1 f(x)\,dx$

$$I = \int 2xf'(x)\,dx = 2xf(x) - \int f(x)\,2\,dx = 2xf(x) - 2\int f(x)\,dx \Rightarrow \int_0^1 2xf'(x)\,dx = [2xf(x)]_0^1 - 2\int_0^1 f'(x)\,dx$$

$$\begin{cases} 2x = u \Rightarrow 2\,dx = du \\ f'(x)\,dx = dv \Rightarrow v = \int f'(x)\,dx = f(x) \end{cases}$$

$$1 = [2xf(x)]_0^1 - 2\int_0^1 f'(x)\,dx \Rightarrow 2\int_0^1 f'(x)\,dx = [2xf(x)]_0^1 - 1 \Rightarrow \int_0^1 f'(x)\,dx = \frac{[2xf(x)]_0^1 - 1}{2} \Rightarrow$$

$$\int_0^1 f'(x)\,dx = \frac{1}{2}\cdot\{[2.1.f(1) - 2.0.f(0)] - 1\} \Rightarrow \int_0^1 f'(x)\,dx = \frac{1}{2}\cdot\{[2.0 - 0] - 1\} = -\frac{1}{2}$$

2.A.- Calcular un polinomio de tercer grado ***p(x) = ax³+ bx²+ cx +d***, sabiendo que verifica:

I) tiene un máximo relativo en **x = 1**

II) tiene un punto de inflexión en el punto **(0 , 1)**

III) se verifica $\int_0^1 p(x)\,dx = \frac{5}{4}$

$$\begin{cases} f'(x) = 3ax^2 + 2bx + c \\ f''(x) = 6ax + 2b \end{cases} \Rightarrow \begin{cases} f(0) = 1 \Rightarrow a.0^3 + b.0^2 + c.0 + d = 1 \Rightarrow d = 1 \\ f'(1) = 0 \Rightarrow 3a.1^2 + 2b.1 + c = 0 \Rightarrow 3a + 2b + c = 0 \Rightarrow 3a + c = 0 \\ f''(1) = 0 \Rightarrow 6a.0 + 2b = 0 \Rightarrow 2b = 0 \Rightarrow b = 0 \end{cases}$$

$$\int_0^1 \left(ax^3 + cx + 1\right) dx = \frac{5}{4} \Rightarrow \frac{a}{4}\cdot\left[x^4\right]_0^1 + \frac{c}{2}\cdot\left[x^2\right]_0^1 + \left[x\right]_0^1 = \frac{5}{4} \Rightarrow \frac{a}{4}\cdot\left(1^4 - 0^4\right) + \frac{c}{2}\cdot\left(1^2 - 0^2\right) + (1-0) = \frac{5}{4} \Rightarrow$$

$$\frac{a}{4} + \frac{c}{2} + 1 = \frac{5}{4} \Rightarrow a + 2c + 4 = 5 \Rightarrow \begin{cases} 3a + c = 0 \\ a + 2c = 1 \end{cases} \Rightarrow \begin{cases} -6a - 2c = 0 \\ a + 2c = 1 \end{cases} \Rightarrow -5a = 1 \Rightarrow a = -\frac{1}{5} \Rightarrow$$

$$-\frac{1}{5} + 2c = 1 \Rightarrow 2c = 1 + \frac{1}{5} \Rightarrow 2c = \frac{6}{5} \Rightarrow c = \frac{3}{5} \Rightarrow p(x) = -\frac{1}{5}\cdot x^3 + \frac{3}{5}x + 1$$

3.A.- Dado el sistema de ecuaciones: $\begin{cases} (m-1)x + y + z = 3 \\ mx + (m-1)y + 3z = 2m-1 \\ x + 2y + (m-2)z = 4 \end{cases}$

a) Discutirlo según los distintos valores de ***m***

b) Resolverla cuando sea compatible indeterminado

$a)$

$$|A| = \begin{vmatrix} m-1 & 1 & 1 \\ m & m-1 & 3 \\ 1 & 2 & m-2 \end{vmatrix} = (m-1)^2(m-2) + 3 + 2m - (m-1) - 6(m-1) - m(m-2) \Rightarrow$$

$$|A| = (m^2 - 2m + 1)(m-2) + 3 + 2m - 7m + 7 - m^2 + 2m = m^3 - 2m^2 - 2m^2 + 4m + m - 2 - m^2 - 3m + 10$$

$$|A| = m^3 - 5m^2 + 2m + 8 \Rightarrow m^3 - 5m^2 + 2m + 8 = (m+1)(m^2 - 6m + 8) \Rightarrow m^2 - 6m + 8 = 0 \Rightarrow$$

$$\begin{array}{r|rrrr} & 1 & -5 & 2 & 8 \\ 1 & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & \underline{|0} \end{array}$$

$$\Delta = 36 - 32 = 4 > 0 \Rightarrow m = \frac{6 \pm \sqrt{4}}{2} \Rightarrow \begin{cases} m = 4 \\ m = 2 \end{cases} \Rightarrow$$

$$m^3 - 5m^2 + 2m + 8 = (m+1)(m-4)(m-2)$$

$$Si\ |A| = 0 \Rightarrow m^3 - 5m^2 + 2m + 8 = 0 \Rightarrow (m+1)(m-4)(m-2) = 0 \Rightarrow \begin{cases} m = -1 \\ m = 2 \\ m = 4 \end{cases} \Rightarrow$$

$$\forall m \in \Re - \{-1\ ,\ 2\ ,\ 4\} \Rightarrow |A| \neq 0 \Rightarrow rang(A) = 3 \Rightarrow Sistema\ Compatible\ Deter\min ado$$

$$\Pr imer\ método. - Roche - Frobenius$$

$$|A/B| = |C_1 \quad C_2 \quad B| = \begin{vmatrix} m-1 & 1 & 3 \\ m & m-1 & 2m-1 \\ 1 & 2 & 4 \end{vmatrix}$$

$$|A/B| = 4(m-1)^2 + (2m-1) + 6m - 3(m-1) - 2(2m-1)(m-1) - 4m$$

$$|A/B| = 4(m^2 - 2m + 1) + 2m - 1 + 2m - 3m + 3 - 4m^2 + 4m + 2m - 2$$

$$|A/B| = 4m^2 - 8m + 4 + 7m - 4m^2 = -m + 4 \Rightarrow Si\ |A/B| = 0 \Rightarrow -m + 4 = 0 \Rightarrow m = 4$$

$$\forall m \in \Re - \{4\} \Rightarrow |A/B| \neq 0 \Rightarrow rang(A/B) = 3 \Rightarrow m = 4 \Rightarrow |A/B| = \begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} = 5 \neq 0 \Rightarrow rang(A/B) = 2$$

Continúa el problema 3.A.-

$a)Continuación$

$\Pr imer\ método. - Roche - Frobenius(Continua)$

$$|A/B| = |C_1 \quad C_3 \quad B| = \begin{vmatrix} m-1 & 1 & 3 \\ m & 3 & 2m-1 \\ 1 & m-2 & 4 \end{vmatrix}$$

$$|A/B| = 12(m-1) + 2m - 1 + 3m(m-2) - 9 - 4m - (m-1)(m-2)(2m-1)$$

$$|A/B| = 12m - 12 - 2m - 10 + 3m^2 - 6m - (m^2 - 3m + 2)(2m-1) =$$

$$|A/B| = 3m^2 + 4m - 22 - 2m^3 + m^2 + 6m^2 - 3m - 4m + 2 = -2m^3 + 10m^2 - 3m - 20$$

$$\Rightarrow Si\ |A/B| = 0 \Rightarrow -2m^3 + 10m^2 - 3m - 20 = 0 \Rightarrow 2m^3 - 10m^2 + 3m + 20 = 0 \Rightarrow (2m^2 - 2m - 5)(m-4) = 0$$

$$\begin{array}{r|rrrr} & 2 & -10 & 3 & 20 \\ 4 & & 8 & -8 & -20 \\ \hline & 2 & -2 & -5 & \underline{|0} \end{array} \qquad \Delta = 4 - 40 = 44 > 0 \Rightarrow m = \frac{2 \pm \sqrt{44}}{4} \Rightarrow \begin{cases} m = \dfrac{1+\sqrt{11}}{2} \\ m = \dfrac{1-\sqrt{11}}{2} \end{cases} \Rightarrow$$

$$\forall m \in \Re - \left\{ \frac{1-\sqrt{11}}{2}\ ,\ 4\ ,\ \frac{1+\sqrt{11}}{2} \right\} \Rightarrow |A/B| \neq 0 \Rightarrow rang(A/B) = 3 \Rightarrow$$

$$m = \begin{cases} 4 \\ \dfrac{1-\sqrt{11}}{2} \\ \dfrac{1+\sqrt{11}}{2} \end{cases} \Rightarrow |A/B| = \begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix} = -2 \neq 0 \Rightarrow rang(A/B) = 2(como\ ejemplo)$$

$$|A/B| = |C_2 \quad C_3 \quad B| = \begin{vmatrix} 1 & 1 & 3 \\ m-1 & 3 & 2m-1 \\ 2 & m-2 & 4 \end{vmatrix}$$

$$|A/B| = 12 + 2(2m-1) + 3(m-1)(m-2) - 18 - 4(m-1) - (2m-1)(m-2)$$

$$|A/B| = 12 + 4m - 2 + 3(m^2 - 3m + 2) - 18 - 4m + 4 - (2m^2 - 4m - m + 2)$$

$$|A/B| = -4 + 3m^2 - 9m + 6 - 2m^2 + 4m + m - 2 = m^2 - 4m = m(m-4) \Rightarrow$$

$$|A/B| = 0 \Rightarrow \begin{cases} m = 0 \\ m = 4 \end{cases} \Rightarrow \forall m \in \Re - \{0\ ,\ 4\} \Rightarrow |A/B| \neq 0 \Rightarrow rang(A/B) = 3 \Rightarrow$$

$$m = \begin{cases} 0 \\ 4 \end{cases} \Rightarrow \begin{vmatrix} 1 & 3 \\ 3 & 7 \end{vmatrix} = -2 \neq 0 \Rightarrow rang(A/B) = 2$$

$Cuando\ m = 4 \Rightarrow rang(A) = rang(A/B) = 2 < Número\ de\ incognitas \Rightarrow Sist.\ Compat.\ In\det er\min ado$

$Cuando\ m = -1 \Rightarrow rang(A) = 2 \neq rang(A/B) = 3 \Rightarrow Sistema.\ Incompatible$

$Cuando\ m = 2 \Rightarrow rang(A) = 2 \neq rang(A/B) = 3 \Rightarrow Sistema.\ Incompatible$

Continúa el problema 3.A.-

Otra forma de realizar el apartado a) es utilizando la reducción por Gauss

$$a)\text{Continuación}$$
$$\text{Segundo método.} - \text{Gauss} \Rightarrow$$
$$\text{Con } m=-1$$
$$A=\left(\begin{array}{ccc|c} -2 & 1 & 1 & 3 \\ -1 & -2 & 3 & -3 \\ 1 & 2 & -3 & 4 \end{array}\right) \equiv \left(\begin{array}{ccc|c} -2 & 1 & 1 & 3 \\ 2 & 4 & -6 & 6 \\ 2 & 4 & -6 & 8 \end{array}\right) \equiv \left(\begin{array}{ccc|c} -2 & 1 & 1 & 3 \\ 0 & 5 & -5 & 9 \\ 0 & 5 & -5 & 11 \end{array}\right) \equiv \left(\begin{array}{ccc|c} -2 & 1 & 1 & 3 \\ 0 & 5 & -5 & 9 \\ 0 & 0 & 0 & 2 \end{array}\right) \Rightarrow$$
$$\text{Sistema Incompatible}$$
$$\text{Con } m=2$$
$$A=\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 3 \\ 1 & 2 & 0 & 4 \end{array}\right) \equiv \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & 1 & -3 \\ 0 & 1 & -1 & 1 \end{array}\right) \equiv \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & -2 \end{array}\right) \Rightarrow \text{Sistema Incompatible}$$
$$\text{Con } m=4$$
$$A=\left(\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 4 & 3 & 3 & 7 \\ 1 & 2 & 2 & 4 \end{array}\right) \equiv \left(\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 12 & 9 & 9 & 21 \\ 3 & 6 & 6 & 12 \end{array}\right) \equiv \left(\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 5 & 5 & 9 \\ 0 & 5 & 5 & 9 \end{array}\right) \equiv \left(\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 5 & 5 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \text{Sist. Compat. Indeterminado}$$
$$b)$$
$$\text{Por Gauss}$$
$$5y+5z=9 \Rightarrow 5y=9-5z \Rightarrow y=\frac{9-5z}{5} \Rightarrow 3x+\frac{9-5z}{5}+z=3 \Rightarrow 15x+9-5z+5z=15 \Rightarrow$$
$$15x=6 \Rightarrow x=\frac{2}{5} \Rightarrow \text{Solución}\left(\frac{2}{5}, \frac{9}{5}-\lambda, \lambda\right)$$
$$\text{Por Rouche}$$
$$\begin{cases} 3x+y+z=3 \\ 4x+3y+3z=7 \end{cases} \Rightarrow \begin{cases} -9x-3y-3z=-9 \\ 4x+3y+3z=7 \end{cases} \Rightarrow -5x=-2 \Rightarrow x=\frac{2}{5} \Rightarrow 3\cdot\frac{2}{5}+y+z=3 \Rightarrow$$
$$6+5y+5z=15 \Rightarrow 5y=9-5z \Rightarrow y=\frac{9-5z}{5}=\frac{9}{5}-z \Rightarrow \text{Solución}\left(\frac{2}{5}, \frac{9}{5}-\lambda, \lambda\right)$$

Evidentemente hay que elegir uno de los dos procedimientos, el segundo valdrá para comprobar

4.A.- Dado el punto **P(1 , 3 , -1)**, se pide:

a) Escribir la ecuación que deben de verificar los puntos **X(x , y , z)** cuya distancia a **P** sea igual a **3**

b) Calcular los puntos de la recta $\begin{cases} x = 3\lambda \\ y = 1+\lambda \\ z = 1-4\lambda \end{cases}$ cuya distancia a **P** es igual a **3**

$a)$

$$3 = \pm\sqrt{(x-1)^2 + (y-3)^2 + (z+1)^2} \Rightarrow 9 = (x-1)^2 + (y-3)^2 + (z+1)^2 \Rightarrow$$

$$9 = x^2 - 2x + 1 + y^2 - 6y + 9 + z^2 + 2z + 1 \Rightarrow x^2 + y^2 + z^2 - 2x - 6y + 2z + 2 = 0 \Rightarrow \textit{Una esfera}$$

$b)$

$$3 = \pm\sqrt{(3\lambda-1)^2 + (1+\lambda-3)^2 + (1-4\lambda+1)^2} \Rightarrow 9 = (3\lambda-1)^2 + (\lambda-2)^2 + (2-4\lambda)^2 \Rightarrow$$

$$9 = 9\lambda^2 - 6\lambda + 1 + \lambda^2 - 4\lambda + 4 + 4 - 16\lambda + 16\lambda^2 \Rightarrow 26\lambda^2 - 26\lambda = 0 \Rightarrow \lambda^2 - \lambda = 0 \Rightarrow (\lambda-1)\lambda = 0 \Rightarrow$$

$$\begin{cases} \lambda = 0 \Rightarrow P_1 \begin{cases} x = 3.0 = 0 \\ y = 1+0 = 1 \\ z = 1-4.0 = 0 \end{cases} \Rightarrow P_1(0\ ,1\ ,0) \\ \lambda - 1 = 0 \Rightarrow \lambda = 1 \Rightarrow P_2 \begin{cases} x = 3.1 = 3 \\ y = 1+1 = 2 \\ z = 1-4.1 = -3 \end{cases} \Rightarrow P_2(3\ ,2\ ,-3) \end{cases}$$

OPCIÓN B

1.B.- a)Resolver el sistema: $\begin{cases} x+2y+3z=1 \\ 2x+y-z=2 \end{cases}$

b) Hallar dos constantes ay b de manera que al añadir al sistema anterior una tercera ecuación : **5x + y + az = b** el sistema resultante sea compatible indeterminado

$a)$

$$\begin{cases} x+2y+3z=1 \\ 6x+3y-3z=6 \end{cases} \Rightarrow 7x+5y=7 \Rightarrow 7x=7-5y \Rightarrow x=\frac{7-5y}{7}=1-\frac{5}{7}y \Rightarrow 1-\frac{5}{7}y+2y+3z=1 \Rightarrow$$

$$3z=\frac{5}{7}y-2y \Rightarrow 3z=-\frac{9}{7}y \Rightarrow z=-\frac{3}{7}y \Rightarrow Solución\left(1-\frac{5}{7}\lambda\ ,\ \lambda\ ,\ -\frac{3}{7}\lambda\right)$$

$b)$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 5 & 1 & \alpha & \beta \end{array}\right) \equiv \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -7 & 0 \\ 0 & -9 & \alpha-15 & \beta-5 \end{array}\right) \equiv \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -7 & 0 \\ 0 & 0 & \alpha-15+21 & \beta-5 \end{array}\right) \Rightarrow \begin{cases} \alpha+6 \Rightarrow \alpha=-6 \\ \beta-5=0 \Rightarrow \beta=5 \end{cases}$$

2.B.-Hallar una matriz **X** tal que **$A^{-1}XA = B$**, siendo $A = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix}$ y $B = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$

$$AA^{-1}XA = AB \Rightarrow IXA = AB \Rightarrow XAA^{-1} = ABA^{-1} \Rightarrow XI = ABA^{-1} \Rightarrow X = ABA^{-1}$$

$$\exists A^{-1} \Rightarrow |A| \neq 0 \Rightarrow |A| = \begin{vmatrix} 3 & 1 \\ -2 & -1 \end{vmatrix} = -3+2 = -1 \neq 0 \Rightarrow A^{-1} = \frac{1}{|A|} \cdot adj\left(A^t\right) \Rightarrow A^t = \begin{pmatrix} 3 & -2 \\ 1 & -1 \end{pmatrix} \Rightarrow$$

$$adj\left(A^t\right) = \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{(-1)} \cdot \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} \Rightarrow$$

$$X = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 9 & 11 \\ -6 & -7 \end{pmatrix}$$

Comprobación

$$A^{-1}XA = B \Rightarrow \begin{pmatrix} 1 & 1 \\ -2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 9 & 11 \\ -6 & -7 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = B \; (Comprobado)$$

3.B.- Calcular los siguientes límites:

$$a)\ \lim_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$$

$$b)\ \lim_{x\to\infty} x\left[arc\ tg\left(e^x\right)-\frac{\pi}{2}\right]$$

$$a)\ \lim_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)=\infty-\infty=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)\left(\sqrt{x^2+x}+\sqrt{x^2-x}\right)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=$$

$$=\lim_{x\to\infty}\frac{x^2+x-\left(x^2-x\right)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\lim_{x\to\infty}\frac{x^2+x-x^2+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\frac{\infty}{\infty}=$$

$$=\lim_{x\to\infty}\frac{\frac{2x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}}=\lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}}}=\frac{2}{\sqrt{1+\frac{1}{\infty}}+\sqrt{1-\frac{1}{\infty}}}=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}=\frac{2}{2}=1$$

$$b)\ \lim_{x\to\infty} x\left[arc\ tg\left(e^x\right)-\frac{\pi}{2}\right]=\infty.0=\lim_{x\to\infty}\frac{arc\ tg\left(e^x\right)-\frac{\pi}{2}}{\frac{1}{x}}=\frac{0}{0}=\xrightarrow{\text{Utilizando L'Hopital}}=\lim_{x\to\infty}\frac{\frac{e^x}{1+e^{2x}}}{-\frac{1}{x^2}}=$$

$$=-\left(\lim_{x\to\infty}\frac{x^2e^x}{1+e^{2x}}\right)=\frac{\infty}{\infty}=\xrightarrow{\text{Utilizando L'Hopital}}=-\left(\lim_{x\to\infty}\frac{2xe^x+x^2e^x}{2e^{2x}}\right)=-\left(\lim_{x\to\infty}\frac{xe^x\left(2+x\right)}{2e^{2x}}\right)=-\left(\lim_{x\to\infty}\frac{x\left(2+x\right)}{2e^x}\right)=$$

$$=\frac{\infty}{\infty}=\xrightarrow{\text{Utilizando L'Hopital}}=-\left(\lim_{x\to\infty}\frac{2+x+1}{2e^x}\right)=\frac{\infty}{\infty}=\xrightarrow{\text{Utilizando L'Hopital}}=-\left(\lim_{x\to\infty}\frac{1}{2e^x}\right)=-\frac{1}{\infty}=0$$

4.B.-Dadas las rectas: $r:\dfrac{x-1}{2}=\dfrac{y-1}{3}=\dfrac{z-1}{4}\quad y\quad s:\dfrac{x+1}{1}=\dfrac{y-2}{-1}=\dfrac{z}{2}$

a) Hallar la ecuación de la recta ***t*** que corta a las dos y es perpendicular a ambas

b) Calcular la mínima distancia entre ***r*** y ***s***

a) El vector director de la recta t, que arranca de puntos generales de r y s, es perpendicular a estas, por ello los productos escalares de los vectores directores (el de t con cada uno de los de r y s) son nulos. Hallado el punto de corte con r o s tendremos definida la recta t.

$$\begin{cases} r:\begin{cases} x=1+2\lambda \\ y=1+3\lambda \\ z=1+4\lambda \end{cases} \\ s:\begin{cases} x=-1+\mu \\ y=2-\mu \\ z=2\mu \end{cases} \end{cases} \Rightarrow \begin{cases} \overrightarrow{v_t}=[1+2\lambda-(-1+\mu),1+3\lambda-(2-\mu),1+4\lambda-2\mu] \\ \overrightarrow{v_r}=(2,3,4) \\ \overrightarrow{v_s}=(1,-1,2) \end{cases} \Rightarrow$$

$$\begin{cases} \overrightarrow{v_t}=(2\lambda-\mu+2,3\lambda+\mu-1,4\lambda-2\mu+1) \\ \overrightarrow{v_r}=(2,3,4) \\ \overrightarrow{v_s}=(1,-1,2) \end{cases} \Rightarrow \begin{cases} \overrightarrow{v_t}\perp\overrightarrow{v_r}\Rightarrow\overrightarrow{v_t}.\overrightarrow{v_r}=0 \\ \overrightarrow{v_t}\perp\overrightarrow{v_s}\Rightarrow\overrightarrow{v_t}.\overrightarrow{v_s}=0 \end{cases} \Rightarrow$$

$$\begin{cases} (2\lambda-\mu+2,3\lambda+\mu-1,4\lambda-2\mu+1)\cdot(2,3,4)=0 \\ (2\lambda-\mu+2,3\lambda+\mu-1,4\lambda-2\mu+1)\cdot(1,-1,2)=0 \end{cases} \Rightarrow \begin{cases} 4\lambda-2\mu+4+9\lambda+3\mu-3+16\lambda-8\mu+4=0 \\ 2\lambda-\mu+2-3\lambda-\mu+1+8\lambda-4\mu+2=0 \end{cases} \Rightarrow$$

$$\begin{cases} 29\lambda-7\mu+5=0 \\ 7\lambda-6\mu+5=0 \end{cases} \Rightarrow \left(\begin{array}{cc|c} 29 & -7 & -5 \\ 203 & -174 & -145 \end{array}\right)=\left(\begin{array}{cc|c} 29 & -7 & -5 \\ 0 & -125 & -110 \end{array}\right) \Rightarrow -125\mu=-110 \Rightarrow \mu=\frac{110}{125}=\frac{22}{25}$$

$$7\lambda-6\cdot\frac{22}{25}=-5 \Rightarrow 7\lambda=-5+\frac{132}{25}=\frac{-125+132}{25}=\frac{7}{25} \Rightarrow \lambda=\frac{1}{25}$$

$$\begin{cases} R\begin{cases} x=1+2\cdot\frac{1}{25}=\frac{27}{25} \\ y=1+3\cdot\frac{1}{25}=\frac{28}{25} \\ z=1+4\cdot\frac{1}{25}=\frac{29}{25} \end{cases} \\ \overrightarrow{v_t}=\left(2\cdot\frac{1}{25}-\frac{22}{25}+2,3\cdot\frac{1}{25}+\frac{22}{25}-1,4\cdot\frac{1}{25}-2\cdot\frac{22}{25}+1\right)=\left(\frac{2-22+50}{25},\frac{3+22-25}{25},\frac{4-44+25}{25}\right) \end{cases} \Rightarrow$$

$$\begin{cases} R\left(\frac{27}{25},\frac{28}{25},\frac{29}{25}\right)\begin{cases} x=1+2\cdot\frac{1}{25}=\frac{27}{25} \\ y=1+3\cdot\frac{1}{25}=\frac{28}{25} \\ z=1+4\cdot\frac{1}{25}=\frac{29}{25} \end{cases} \\ \overrightarrow{v_t}=\left(\frac{30}{25},\frac{0}{25},\frac{15}{25}\right)=\left(\frac{6}{5},0,\frac{3}{5}\right)\equiv(6,0,3)\equiv(2,0,1) \end{cases} \Rightarrow t:\begin{cases} x=\frac{27}{25}+2\beta \\ y=\frac{28}{25} \\ z=\frac{29}{25}+\beta \end{cases}$$

Continúa el problema 4.B.-

b) La mínima distancia entre las rectas r y s es la que existe entre los puntos de corte, R y S, de la perpendicular hallada con cada una de ellas, nos servirá `l` y `m` halladas para buscarlos

$$\begin{cases} R\left(\frac{27}{25}, \frac{28}{25}, \frac{29}{25}\right) \\ S\begin{cases} x = -1 + \frac{22}{25} = -\frac{3}{25} \\ y = 2 - \frac{22}{25} = \frac{3}{25} \quad \Rightarrow R\left(-\frac{3}{25}, \frac{3}{25}, \frac{44}{25}\right) \\ z = 2 \cdot \frac{22}{25} = \frac{44}{25} \end{cases} \end{cases} \Rightarrow$$

$$d_{rs} = d_{RS} = \sqrt{\left[\frac{27}{25} - \left(-\frac{3}{25}\right)\right]^2 + \left(\frac{28}{25} - \frac{3}{25}\right)^2 + \left(\frac{29}{25} - \frac{44}{25}\right)^2} = \sqrt{\left(\frac{30}{25}\right)^2 + \left(\frac{25}{25}\right)^2 + \left(-\frac{15}{25}\right)^2}$$

$$d_{rs} = \sqrt{\left(\frac{6}{5}\right)^2 + 1^2 + \left(-\frac{3}{5}\right)^2} = \sqrt{\frac{36}{25} + 1 + \frac{9}{25}} = \sqrt{\frac{36 + 25 + 9}{25}} = \sqrt{\frac{70}{25}} = \frac{\sqrt{70}}{5}\ u$$