Soluciones

1. a) $f'(1) = \lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1} = \lim_{x\to 1}\dfrac{2x^2+x+3-6}{x-1} =$

$= \lim_{x\to 1}\dfrac{(x-1)(2x+3)}{x-1} = 5$

b) $f'(-2) = \lim_{x\to -2}\dfrac{f(x)-f(-2)}{x+2} = \lim_{x\to -2}\dfrac{\frac{3}{x+5}-1}{x+2} =$

$= \lim_{x\to -2}\dfrac{-(x+2)}{(x+5)(x+2)} = -\dfrac{1}{3}$

2. a) $v_m = \dfrac{s(2)-s(0)}{2-0} = \dfrac{14}{2} = 7$ m/s

b) $v_i = \lim_{h\to 0}\dfrac{s(1+h)-s(1)}{h} = \lim_{h\to 0}(4h+7) = 7$ m/s

3. $m = f'(-2) = \lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h} =$

$= \lim_{h\to 0}\dfrac{\left[2(2+h)^2-3(2+h)\right]-(2\cdot 2^2-3\cdot 2)}{h} = 5$

$y - f(2) = m(x-2) \Rightarrow y - 2 = 5(x-2)$

$\operatorname{tg}\alpha = 5 \Rightarrow \operatorname{arctg}(5) \approx 78°\ 41'\ 24''$

4. $f(0) = 1$, $f'(x) = \dfrac{-1}{(x+1)^2}$, $f'(0) = -1$; la ecuación de la recta tangente es $y - 1 = -1\cdot(x-0) \Rightarrow y = -x+1$.

Punto de corte: $\begin{cases} y = -x+1 \\ y = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 0 \end{cases} \Rightarrow P(1, 0)$

5. a) $(f+g)'(3) = f'(3) + g'(3) = 0 + (-5) = -5$

b) $(f\cdot g)'(-2) = f'(-2)\cdot g(-2) + f(-2)\cdot g'(-2) =$
$= 6\cdot(-1) + 3\cdot 7 = -6 + 21 = 15$

c) $\left(\dfrac{f}{g}\right)'(-2) = \dfrac{f'(-2)\cdot g(-2) - f(-2)\cdot g'(-2)}{(g(-2))^2} =$

$= \dfrac{6\cdot(-1) - 3\cdot 7}{(-1)^2} = \dfrac{-6-21}{1} = -7$

d) $(f\circ g)'(-2) = f'(g(-2))\cdot g'(-2) = f'(-1)\cdot g'(-2) =$
$= (-3)\cdot 7 = -21$

e) $(g\circ f)'(-2) = g'(f(-2))\cdot f'(-2) = g'(3)\cdot f'(-2) =$
$= (-5)\cdot 6 = -30$

6. $f'(x) = 2$ $\qquad g'(x) = \dfrac{1}{2\sqrt{x}}$ $\qquad h'(x) = 2x$

a) $(g\circ h)'(2) = g'(h(2))\cdot h'(2) = \dfrac{1}{2\sqrt{5}}\cdot 4 = \dfrac{2}{\sqrt{5}}$

b) $(h\circ g\circ f)'(1) = h'(g(f(1)))\cdot g'(f(1))\cdot f'(1) =$

$= 2\sqrt{3}\cdot\dfrac{1}{2\sqrt{3}}\cdot 2 = 2$

c) $(f\circ h\circ g)'(4) = f'(h(g(4)))\cdot h'(g(4))\cdot g'(4) =$

$= 2\cdot 2\sqrt{4}\cdot\dfrac{1}{2\sqrt{4}} = 2$

d) $(g\circ f\circ h)'(x) = \dfrac{1}{2\sqrt{1+2(x^2+1)}}\cdot 2\cdot 2x = \dfrac{2x}{\sqrt{2x^2+3}}$

7. a) $f'(x) = 5(x^2-3x+5)^4\cdot(2x-3)$

b) $g'(x) = 2\operatorname{sen}(\ln(2x+1))\cdot\cos(\ln(2x+1))\cdot\dfrac{2}{2x+1}$

c) $h'(x) = \dfrac{1}{2\sqrt{\cos(1-3x)}}\cdot(-\operatorname{sen}(1-3x))\cdot(-3) =$

$= \dfrac{3\operatorname{sen}(1-3x)}{2\sqrt{\cos(1-3x)}}$

8. $f(c) = -1 \Rightarrow c^3 + c - 11 = -1 \Rightarrow c = 2$

$f'(x) = 3x^2 + 1 \Rightarrow f'(2) = 13$

$(f\circ f^{-1})(x) = x$. Derivando la función compuesta:

$f'(f^{-1})(x))\cdot(f^{-1})'(x) = 1 \Rightarrow$

$\Rightarrow (f^{-1})(-1) = \dfrac{1}{f'(f^{-1}(-1))} = \dfrac{1}{f'(2)} = \dfrac{1}{13}$

9. a) $\ln f(x) = \dfrac{1}{x}\cdot\ln(2x+1) \Rightarrow$

$\Rightarrow \dfrac{f'(x)}{f(x)} = -\dfrac{1}{x^2}\cdot\ln(2x+1) + \dfrac{1}{x(2x+1)} \Rightarrow$

$\Rightarrow f'(x) = \sqrt[x]{2x+1}\cdot\left(-\dfrac{\ln(2x+1)}{x^2} + \dfrac{1}{x(2x+1)}\right)$

b) $\ln(g(x)) = (2x+1)\cdot\ln(\operatorname{sen} x) \Rightarrow$

$\Rightarrow \dfrac{g'(x)}{g(x)} = 2\ln(\operatorname{sen} x) + (2x+1)\dfrac{\cos x}{\operatorname{sen} x} \Rightarrow$

$\Rightarrow g'(x) = (\operatorname{sen} x)^{2x+1}\cdot(2\ln(\operatorname{sen} x) + (2x+1)\operatorname{cotg} x)$

c) $\ln(h(x)) = 5x^2\ln 3 \Rightarrow \dfrac{h'(x)}{h(x)} = (10\ln 3)x \Rightarrow$

$\Rightarrow h'(x) = 3^{5x^2}(10\ln 3)x$

10. $2x + 3y + 3xy' + 2yy' = 0 \Rightarrow y' = \dfrac{-2x-3y}{3x+2y}$

$f'(2, -1) = \dfrac{-4+3}{6-2} = -\dfrac{1}{4}$; $y + 1 = -\dfrac{1}{4}(x-2)$

11. $dy = \dfrac{3}{2\sqrt{3x-2}}dx \Rightarrow dy(x=9) = \dfrac{3}{2\cdot 5}\cdot 0{,}2 = 0{,}06.$

12. $y = \sqrt[3]{x} \Rightarrow dy = \dfrac{1}{3\sqrt[3]{x^2}}dx$

$\sqrt[3]{345} = \sqrt[3]{343} + \Delta y \approx 7 + dy =$

$= 7 + \dfrac{1}{3\sqrt[3]{343^2}}\cdot 2 \approx 7{,}0136$

13. a) $dy = (10x-7)dx$ $\quad$ b) $ds = \cos t\, dt$ $\quad$ c) $du = \dfrac{1}{v}dv$

14. $dV = 4\pi r^2 dr$. Representa el volumen de una "superficie" esférica de radio r y espesor dr.